**Investigating representations of ratio among prospective mathematics and science teachers: an international study**

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The importance of teachers’ content knowledge *for* teaching is acknowledged as a factor of teacher quality in mathematics and science. At the 2011 annual conference of ATEE in Riga, the Science and Mathematics Education RDC initiated a study of our prospective teachers’ content knowledge of *ratio* for teaching mathematics and science using a grounded theory design. An analysis of the meanings and representations offered by the participants indicated emergent themes leading to the following conjectures. Participants who associated meanings that reflect two variables while providing many, varied, and relevant representations possessed relational understanding of ratio (Skemp 1976). The RDC proposes how further study with other participants will begin the process of establishing the validity of these conjectures.

**Introduction**

# A major goal of the ATEE Research and Development Centre (RDC) ‘Science and Mathematics Education’ is to conduct international research studies that address significant problems related to the preparation of prospective teachers and the professional development of classroom teachers. At times the studies include both primary and secondary mathematics and science teachers, and at other times the RDC members work on problems in their own disciplines. Previously we studied the content knowledge and pedagogical content knowledge held by prospective teachers in five different countries concerning concepts relevant to both mathematics and science teaching (for example Berenson et al. 1997; Frederik et al. 1999; Oldham et al. 2000; Van Driel, De Jong, & Verloop 2002). While the annual ATEE meetings provide face-to-face meetings for the research group, e-mail is a major communication tool during the year. Considerable time is spent during the annual meeting in our RDC deciding on a significant problem relevant in our varied contexts of teacher education. It is necessary that the parameters of the problem allow for data collection in a variety of settings. Rather than conducting comparative studies in different settings, we examine our data for similarities across countries and universities. This leads to greater understanding and a base of knowledge of mathematics and science teacher education within the international education community about mathematics and science for teaching.

# This paper describes the first phase of a new RDC project on investigating prospective teachers’ knowledge of the concept of *ratio*, a key idea in both mathematics and science education. In the following section of the paper, the background to the project and choice of topic is described and research questions are stated. The methodology, implementing a grounded theory design, is then outlined. Initial findings are presented and emergent themes identified. The discussion that follows proposes the conjectures taken from the emergent themes and provides supporting evidence from existing research of the viability of these conjectures. Finally, the RDC proposes how these conjectures will be tested with other participants to begin the process of establishing their validity.

**Investigating meanings and representations of ratio: beginning the project**

The importance of teachers’ content knowledge *for* teaching is acknowledged by many as a factor of teacher quality in mathematics and science. Hill and Ball (2004), among others, advocate that it is important for teachers to understand deeply the discipline concepts for their levels of teaching *and* the associated methodology for teaching those concepts. Research with regard to both mathematics and science teaching suggests that teachers’ knowledge of the discipline, in particular knowledge of the subject-specific methodology in relation to teaching, emerges as a key factor in student learning (Darling-Hammond 2000). As teacher educators who prepare prospective teachers for primary and secondary teaching, we recognize this important link between what teachers know and what their students come to understand about fundamental principles of mathematics and science.

At the 2011 annual meeting of ATEE in Riga, our RDC coalesced around a research problem related to our prospective teachers’ knowledge of ratio *for* teaching mathematics and science at primary and secondary levels. There are a number of definitions for ratio but clearly, no one definition has emerged that satisfies all educators (Lamon 2007). Clark, Berenson, and Cavey (2003) report on the multiple models held by mathematics educators and propose a model that situates part-whole relationships in the intersection between ratio and fraction. Our approach to defining ratio is to view the term as a comparison of like or unlike elements. While we include rates in the category of ratio, we do not include fractions that express part-whole relationships.

Ratio is an important concept in many middle and secondary school curricula. For example, probability, slope, trigonometry, and the derivative are just a few examples of mathematical concepts that use ratio as a tool. Mixture, solutions, moles, simple machines, and acceleration are some concepts in the physical sciences that depend on ratio and proportional reasoning; proportional thinking is relevant also in the use of chemical formulas and equations and photosynthesis rate. Clearly ratio is a key tool to proportional reasoning and fundamental in terms of higher order thinking and problem solving in multiple mathematics and science content areas (Vergnaud 1997).

Difficulties associated with the ratio concept are documented in the literature. Researchers have studied the problem from a number of perspectives, contexts, and subjects. Interest in Piaget and Inhelder’s work on children’s reasoning became a focus of science education research beginning in the early sixties (eg. Lawson 1986). In general, both science and mathematics educators focused on children’s ideas of ratio over the intervening years. More recently Livy and Vale (2011) summarised evidence that students in the middle years of schooling have poor understanding of ratio and proportional reasoning. Evidently, little has changed over the past 50 years in terms of children’s understanding.

The intersection of ratio between mathematics and science concepts makes it an interesting area of study of teacher knowledge, not only of teacher content knowledge but also of methodological knowledge. Recently Livy and Vale (2011) reported that 297 prospective teachers gave low levels of correct responses to relevant ratio and proportion test items. The implications of such findings are worrying. However, few studies seem to focus on teacher education and few have examined teachers’ knowledge of ratio *for* teaching (Ball, Lubienski, & Mewborn 2001). Since ratio is such a valuable tool of proportional reasoning in mathematics and science, it is useful to study *representations* that teachers associate with ratio across the two disciplines.

Representations are defined here as any ideas associated with another idea in mathematics or science that is written, drawn, or spoken. A prospective science teacher may recall using ratios to draw a diagram of a lever or a description of a lab to measure speed. A prospective mathematics teacher may associate ratio with slope on a graph or the scale on a map or the odds for dice. Representations are useful tools for researchers in mathematics and science education to study students and teachers’ ideas (see for example Janvier 1987; van Someren 1998; Arcavi 2003; Lee, & Luft 2008). It is with the study of teachers’ representations of ratio that we began our study using a *grounded theory* research design (Strauss, & Corbin 1990).

A grounded theory approach seeks to develop conjectures derived from a given sample of participants. The literature of published research is studied to find evidence for continuing study of the conjectures. These are then tested and retested with different samples and/or populations of participants (for example Birks 2011). In this initial phase of our study we chose to study the following questions.

a) What meanings do prospective teachers at primary and secondary levels in Ireland, Portugal and the USA give to the term ‘ratio’?

b) What multiple representations do these prospective teachers associate with the term ‘ratio’?

c) Do the prospective teachers’ descriptive meanings and representations indicate different levels of understanding for teaching ratio?

Once the questions were identified, we agreed on a methodology that would accommodate our various settings, research control boards of human subjects, and participants. This is described in the next section.

**Methodology**

As indicated above, the four authors met together during the Riga ATEE Conference, in 2011; they designed the research project, selected the data collection technique and prepared the data collection instrument. It was decided to collect data from classes made up of, or attended chiefly by, prospective teachers in their four institutions, as these were easily available to researchers and were judged appropriate for fulfilling the research objectives settled for this paper. Some of the classes catered chiefly for prospective primary teachers, for whom mathematics and science education were only two strands among many in their teacher education programmes. Other classes catered for prospective secondary teachers, chiefly those choosing mathematics or science as a major or minor second-level teaching subject.

As data were to be collected in three different countries (two English-speaking but still with some English language variations, and a Portuguese-speaking country), the authors chose the questionnaire technique. This makes it easier to minimize language differences than conducting interviews would do.

Then, the authors prepared a questionnaire with five items focusing on the ratio concept:

1. What does the term ratio means to you?

2a. When do you use ratios?

2b. Who else uses ratios?

3. How do you represent a ratio using mathematical symbols?

4. Draw several representations of how ratios are used

Afterwards, three versions (for USA, Ireland and Portugal) were prepared. The questionnaires were anonymous but they ask respondents to give some information on the school level and the school subject that they were preparing to teach. To facilitate administration, the questions were arranged on a sheet of A4 (European) or Letter size (USA) paper as shown in Fig. 1. It was envisaged that they could be completed in a short period, say ten minutes at the end of a lecture. This would not unduly disrupt the running of the class.

**Fig. 1:** Layout of the questionnaire

After obtaining permission from the relevant board in each of the four institutions, the questionnaire was administered in the selected education classes. Administration was carried out by the relevant researcher or by a trained colleague who was given the appropriate instructions.

Questionnaires were received from 171 respondents. However, thirteen Portuguese students took the word *‘razão’* in its everyday rather than its mathematical sense; *‘ter razão’* means to be correct or to put forward correct arguments. These respondents – chiefly prospective science teachers rather than mathematics teachers – were excluded, and 158 questionnaires were analysed. The distributions across the school level at which the respondents were preparing to teach, their main subject areas (where relevant), their professional stage, and the country in which they were studying are shown in Table 1. The data are presented in order to give an overall picture of the responding cohort; however, other than noting some variations that may reflect other differences in understanding or culture between students in different countries, analysis by sub-category is outside the scope of this paper.

As far as data analysis is concerned, data from each institution were examined by the relevant author. Responses (converted into English when necessary) were tabulated for each question and codes were devised. Afterwards, codes were entered into spreadsheets which were shared among the authors and adjusted to facilitate comparability of answers among countries. Then, relative frequencies per question and category (code) were calculated. In addition, interesting responses (that is answers that include quite deep explanations) were examined in more detail so that emergent themes were identified. Addressing the research questions for this paper, reporting is restricted to data from questions 1, 3 and 4.

**Table 1**: Characteristics of the achieved sample

|  |  |  |  |
| --- | --- | --- | --- |
| **Category** | **Sub-category** | **#** | **Total for category** |
| School level | Primary | 101 | 158 |
| Secondary\* | 53 |
| Other | 4 |
| *\*Secondary school subject* | *Mathematics* | *14* | *53* |
| *Science* | *32* |
| *Other* | *7* |
| Professional stage | Pre-service | 149 | 158 |
| Other | 9 |
| Country | American | 127 | 158 |
| Irish | 16 |
| Portuguese | 15 |

**Findings**

Responses were very varied, some participants offering rich meanings for ratio and / or multiple representations, while others provided little information. Findings are presented for instrument questions 1, 3 and 4 in turn. They are described here in terms of response clusters that were identified from the coding process and frequency counts. Emergent themes reflecting deeper analysis and comparison with the literature are presented in the following sections of the paper.

***Question 1: What does the term ‘ratio’ mean to you?***

*Response cluster 1.1: Comparison / relationship*

Many responses included mention of comparison or relationship. Participants used words or phrases such as ‘compare’, ’comparison, ‘compared to’ or (to a lesser extent) ‘relationship’, ‘related’. Portuguese participants were less likely than were American or Irish participants to use this category.

*Response cluster 1.2: Fraction / percentage / proportion / splitting*

A second theme reflected in many responses referred to topics typically associated with elementary or junior second level curricula: fractions, percentage, proportion and splitting (as in ‘a bag of 15 sweets is divided between John and Jane in the ratio 2 : 3; how many does each child get?’).

*Response cluster 1.3: Other*

Occasional references were made to rate, scale and odds.

***Question 3: How do you represent a ratio using mathematical symbols?***

*Response cluster 3.1: Use of the colon or equivalent notation*

The most usual response involved the colon (:) notation, either on its own, or in the form \_\_\_: \_\_\_ (such as *x* : *y* or 3 : 2). In some cases, the word ‘to’ replaced the colon. Usually it was the only notation offered (with or without the provision of examples). All the Irish participants included the colon notation in their responses, though one did so incorrectly. This notation was frequently used by the American participants also.

*Response cluster 3.2: Fractions*

Some participants used fraction notation. This was notably prevalent amongst Portuguese participants.

There were few other responses.

***Question 4: Draw several representations of how ratios are used***

Responses here can be divided into two categories, depending on whether or not the participants interpreted the word ‘draw’ literally or chose to provide written or symbolic responses. It should be noted also that several students made no response to this question.

*Response cluster 4.1: Drawings, diagrams and other pictorial representations*

This cluster displayed considerable variety. One form of response reflected numerical comparisons or representations and is illustrated in Fig. 2. Other responses in this figure indicated comparisons of one and two variables usually reflecting an everyday context.

**Fig. 2:** Examples of numerical comparisons and representations

A second form of response – not used by many participants – showed a mathematical diagram or chart, reflecting geometrical properties (for example similarity) or statistical presentation (for example a bar chart highlighting relative heights of bars). Examples are shown in Fig. 3.

**Fig. 3:** Examples of mathematical diagrams and charts

A few participants tried to capture the idea of scale, for example by sketching a map. Finally, some participants provided drawings that illustrated applications of ratio – for example cookery, architecture and design. In some cases the drawings were labelled with numbers or algebraic labels so as to make the ratio aspects explicit.

*Response cluster 4.2: Numerical, algebraic or verbal representations*

The responses here included examples that were similar to those for question 3, for instance of form 2 : 3, 4/5 or ⅜. A few students answered using words, for instance ‘assets : liabilities’.

**Emergent themes**

We answer the research questions with the identification of emergent themes arising from the participants’ accounts of the meanings and representations they ascribe to ratio. Some descriptions, including representations, emphasize or allow us to infer that the participants’ concept includes the notion of two variables; some appear to refer to uses or applications or special types of ratio; and some relate to part-whole relationships. Table 2 shows typical instances of responses illustrating the three themes. The themes can be seen also in the wide range of representations offered in answer to question 4. For example, Fig. 2 above shows instances of pictorial representations with one and with two variables (with or without labels) as well as a typical representation of a part-whole relationship, while Fig. 3 above illustrates comparisons.

As noted in the introduction, we prefer to define ratio in terms of a comparison of like or unlike elements (the first theme), and to exclude the part-whole relationships often reflected in use of fraction notation (the third theme). Thus, we infer that participants who make use of the latter notation – and especially those whose responses did not include any reference to the comparisons revealed in the first theme – *may not have adequate knowledge, or a full understanding*, of the concept of ratio.

**Table 2:** Emergent themes for participants’ descriptions of the meanings they ascribe to ratio

|  |  |  |
| --- | --- | --- |
| **Infers two distinct variables** | **Types / uses / applications of ratio** | **Part / whole relationships** |
| Comparison | Rate | Fraction |
| Relationship | Scale | Decimal |
|  | Odds |  |
|  | Proportion |  |
|  | Division / splitting |  |
|  | Percent |  |

**Discussion**

This study examined the meanings and representation of prospective mathematics and science teachers to determine their knowledge for teaching of the ratio concept. Knowledge and understanding are addressed in a variety of ways in the literature, often in terms of a dichotomy: for example conceptual versus procedural knowledge (Hiebert 1986), relational versus instrumental understanding (Skemp 1976), and knowing ‘why’ versus knowing ‘that’ (Shulman 1986). Both elements in such dichotomies are important (National Research Council 2001). In fact it is more appropriate to think of ‘both … and’ than of ‘either … or.’ However, the first element in each case is of particular value with regard to sense-making and meaning-making, currently key phrases with regard to the implementation of ‘reform’ curricula. Moreover, according to Hiebert and Grouws (2007), a feature of good teaching is that the teacher can highlight connections, or relationships, between different concepts. This guided our decision to employ Skemp’s terminology. In particular, we looked for indicators of *relational understanding*: understanding of ‘why’, of how the concepts involved in ratio are linked to each other and to other mathematical or scientific concepts. Using this language indicates that some prospective teachers have stronger relational understanding of ratio than others. Additionally, the conjectures define the parameters of that relational understanding (See Table 3). The finding infers that these participants will be able to make more meaningful connections between ideas related to proportional reasoning and problem solving for their students.

We may not be able to form conjectures with regard to *instrumental understanding*, or understanding ‘how’ – for example, how to calculate the answer to a particular example involving ratio. Our instrument did not require participants to carry out calculations, so in general did not allow us to draw inferences in this respect. Some responses did include arithmetic or algebraic calculations, mostly correct but some containing errors. However, the majority did not include calculations. It is noted also that the responses to question 3 do not always contribute to the identification of relational understanding. The responses revealed in general that the participants had Shulman’s knowledge ‘that’ (for example, that the colon symbol is used to represent ratio). The form of the question did not require them to expand their answers or to display their relational understanding, though some participants did so.

We point out above that, in responding to the questionnaire, some participants provided multiple meanings or representations. In some cases these were instances of the same basic representation (for example, 2 : 1 and *x* : *y*); in other cases, the representations were fundamentally different. Crowley and Tall (2006, 57), seeking to understand differences in subsequent performance between two students who initially achieved similar test scores, propose a theory suggesting that ‘mental structures … [are] rich and well-connected in those who succeed, but limited and poorly connected in those who eventually fail.’ The successful student ‘demonstrated links between graphical and symbolic representations, as well as links to and between procedures’; the unsuccessful student ‘merely learnt a set of procedures’ which were ‘not organized in a useful way that would allow her to build on them’ (Crowley, & Tall 2006, 64-65). The descriptions highlight the essential difference between relational and instrumental understanding. From examining our data in the light of this work, we conjecture that the participants using more representations, and especially representations of different types, are displaying more relational understanding.

**Table 3:** Conjectured indicators of presence or absence of relational understanding

|  |  |
| --- | --- |
| **Displays relational understanding** | **Does not display relational understanding** |
| Meaning of ratio reflects two variables | Meaning of ratio does not reflect two variables |
| Provides many representations | Provides few representations |
| Uses multiple types of representation | Uses few types of representation |
| Cites / draws relevant applications | Provides symbolic representations only |

A further feature of the responses especially to question 4 was the presence or absence of drawings or other attempts to show applications, rather than (or as well as) repeating symbolic representations from question 3. It can be inferred that familiarity with the contexts in which ratio is used is another indicator of relational understanding. Hence, overall, *we conjecture that the elements listed in Table 3 are likely indicators of whether or not participants have relational understanding*.

As pointed out in the introduction, the aim of our study is to examine our data for *similarities* across sites (such as countries, or universities within countries); this is not a study of between-country differences. However, Nunes and Bryant (1997) assert that mathematics is a cultural invention; moreover, as Nunes (1997, 32) points out, the subject is learnt in the context of cultural practices. Thus, there may be culturally specific elements in our data that need to be identified and borne in mind during the search for similarities. The clearest example above occurs in the case of the Portuguese students who interpreted the Portuguese word for ratio – *razão* – in terms of its everyday, rather than its mathematical meaning. As this occurred chiefly for prospective science teachers rather than prospective mathematics teachers, a subject-specific culture may be operating here. For native speakers of English, the problem did not arise. In the findings section, we draw attention to other instances where cultural practices within a country may well have affected the style and/or scope of the responses, and may mask underlying regularities in students’ understanding. A related point arose when this paper was presented at the ATEE conference in Eskisehir (August 2012). Turkish members of the audience reacted negatively to the representation of ‘one teacher to three students’ (Fig. 2), as the use of labels as illustrated in the figure was regarded, not as evidence of identification of two variables, but as being fundamentally incorrect. Such cultural variations add a layer of challenge to international studies, but we do not regard them as invalidating the findings.

**Conclusion**

One of several limitations of this study stems from the fact that the research on novice teachers indicates that they may use representations and definitions incorrectly in their teaching especially if student questions arise that stray beyond the novice teachers’ scripted lessons (Hogan, Rabinowitz, & Craven 2003). A limitation of the instrument is that the participants in this study may not have taken the time to think deeply about their responses and therefore their ideas were not fully communicated. However, a benefit of the instrument is that it can administered in about 10 minutes of classroom time, a scarce commodity for every teacher educator, and perhaps several of the questions may prove to be viable for use as a quick assessment tool.

Future research plans for the RDC may include the development of an interview protocol to capture more in-depth knowledge for teaching of ratio among prospective mathematics and science teachers. It may prove useful to incorporate lesson planning to enhance the findings and to perhaps investigate whether prospective teachers depend on both instrumental and relational knowledge when planning a lesson. Another approach may study in-service teachers’ meanings and representations of ratio. We have already recruited researchers from other countries to participate in our next investigation with a view to presenting their work at the annual meeting of ATEE in 2013.

**References**

Arcavi, A. 2003. The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics,* no. 52(3): 215-241.

Ball, D.L., Lubienski, S., and Mewborn, D.S. 2001. Research on teaching mathematics: The unsolved problem of teaching mathematics knowledge. In *Handbook of Research on Teaching* (4th ed.), ed. V. Richardson, 433-456. NY: Macmillan.

Berenson, S., van der Valk, T., Oldham, E., Runesson, U., Moreira, C., and Broekman, H. 1997. An international study to investigate prospective teachers’ content knowledge of the area concept. *European Journal of Teacher Education,* no. 20: 137-150.

Birks, M. 2011. *Grounded theory: A practical guide.* Los Angeles: Sage.

Clark, M.R., Berenson, S.B., and Cavey, L.O. 2003. A comparison of ratios and fractions and their roles as tools in proportional reasoning. *Journal of Mathematical Behavior*, no. 22: 29-317.

Crowley, L., and Tall, D. 2006. Two students: why does one succeed and the other fail? In *Retirement as Process and Concept: a Festschrift for Eddie Gray and David Tall, ed.* A. Simpson, 57-65. Durham: School of Education, University of Durham.

Darling-Hammond, L. 2000. Teacher quality and student achievement. *Education Policy Analysis*, no. 8(1). Retrieved from [http://epaa.asu.edu/ojs/article/view/392](https://go.tcd.ie/owa/redir.aspx?C=ad0569b3b33c484283a5a821c4613d5d&URL=http%3a%2f%2fepaa.asu.edu%2fojs%2farticle%2fview%2f392" \t "_blank).

Frederik, I., van der Valk, T., Leite, L., and Thoren, I. 1999. Pre-service teachers and conceptual difficulties on temperature and heat. *European Journal of Teacher Education*, no. 22(1): 61-74.

Hiebert, J. (ed.), 1986. *Conceptual and Procedural Knowledge: the Case of Mathematics*. Hillsdale, NJ: Erlbaum.

Hiebert, J., and Grouws, D. 2007. The effects of classroom mathematics teaching on students’ learning. In *Second Handbook of Research on Mathematics Teaching and Learning*, ed. F.K. Lester, 371-404. Charlotte, NC: Information Age Publishing.

Hogan, T., Rabinowitz, M., & Craven, J.A. 2003. Representation in teaching: Inferences from research of expert and novice teachers. *Educational Psychologist,* no. 38(4): 235-247.

Hill, H.C., and Ball, D.L. 2004. Learning mathematics for teaching: Results from California’s mathematics professional development institutes. *Journal for Research in Mathematics Education,* no. 35(5): 330-351.

Janvier, C. (ed.), 1987. *Problems of representation in the teaching and learning of mathematics.* London: Lawrence Erlbaum.

Lamon, S.J. 2007. Rational numbers and proportional reasoning: Towards a theoretical framework. In *Second Handbook of Research on Mathematics Teaching and Learning*, ed. F.K. Lester*,* 629-668. Charlotte, NC: Information Age Publishing.

Lawson, A.E. 1986. A review of research on formal reasoning and science teaching. *Journal of Research in Science Teaching*, no. 22(7): 569-617.

Lee, E., and Luft, J.A. 2008. Experienced secondary science teachers’ representations of pedagogical content knowledge. *International Journal of Science Education, no.* 30(10): 1343-1363.

Livy, S., and Vale, C. 2011. First year pre-service teachers’ mathematical content knowledge: Methods of solution for a ratio question. *Mathematics Teacher Education and Development*, no. 13 (2): 22-43.

National Research Council. 2001. *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: The National Academies Press.

Nunes, T. 1997. Systems of signs and mathematical reasoning. In. *Learning and Teaching Mathematics: An International Perspective*, ed. T. Nunes and P. Bryant, 29-44. Hove, UK: Psychology Press.

Nunes, T., and Bryant, P. (eds.), 1997. *Learning and Teaching Mathematics: An International Perspective*. Hove, UK: Psychology Press.

Oldham, E., van der Valk, T., Broekman, H., and Berenson, S. 2000. Beginning pre-service teachers’ approaches to teaching the area concept: Identifying tendencies towards realistic, structuralist, mechanist, or empiricist mathematics education*. European Journal of Teacher Education,* no.22(1): 23-43.

Shulman, L.S. 1986. Those who understand: Knowledge growth in teaching. *Educational Researcher*, no. 15(2): 4-14.

Skemp, R. 1976. Relational understanding and instrumental understanding. *Mathematics Teaching,* no. 77: 20-26.

Strauss, A., and Corbin, J. 1990. *Basics of qualitative research: Grounded theory procedures and techniques*. Newbury Park, California: Sage Publications, Inc.

Van Driel, D., De Jong, O., and Verloop, N. 2002. The development of preservice chemistry teachers’ pedagogical content knowledge. *Science Education*, no. 86(4): 572-590.

# Van Someren, M.W. (ed.), 1998. *Learning with multiple representations.* NY: Elsevier Science.

Vergnaud, G. 1997. The nature of mathematical concepts. In *Learning and Teaching Mathematics: An International Perspective*, ed. T. Nunes and P. Bryant, 5-28. Hove, UK: Psychology Press.

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